Warm-up!

Rewrite each as a single logarithm base 5. Your answers should be in the form $\log_5(\text{expression})$.

$1 + \log$

$1\log_5 5 + \log_5(3x) - 3\log_5(x) = \log_5\left(\frac{(3x)(5)}{(x)^3}\right)$

$\log_5\left(\frac{15}{x^2}\right)$

Log Functions Day 2:

- (2.10) Exponents vs Logs Inverse Relationship
- (2.12) More Log Properties
- (2.10/2.11) Recognizing Log Functions From Tables ✓

Homework: Inverse of Exponentials and More Log Properties

$\log_5\left(\frac{(2x^5)^2}{x^5}\right) = \log_5\left(\frac{4x^{10}}{x^5}\right) = \log_5(4x^5)$

Some anticlimactic math problems

A) Use a calculator/Desmos to find $\log_4 17$. ≈ 2.044

B) Use a calculator/Desmos to find 3^{11}. $= 177147$

C) Let A be your answer to question A. Find 4^A. $= 4^{2.044}$

$4^{\log_4 17} = 17$

D) Let B be your answer to question B. Find $\log_3 B$.

$\log_3(177147) = 11$

$\log_3(3^{11}) = 11$

Logs are the inverse functions to exponents

$$f(x) = \log_b x \text{ and } g(x) = b^x \text{ are inverse functions}$$
(where $b > 0$ and $b \neq 1$)

$(f \circ g)(x) =$ $f(g(x)) = \log_b(b^x) = x$

$(g \circ f)(x) =$ $g(f(x)) = b^{\log_b x} = x$

Consequences of the previous page

A) If the point $(2,10)$ is on the graph of $\boxed{y = a^x}$, what point is on the graph of $\boxed{y = \log_a x}$?

$(10, 2)$

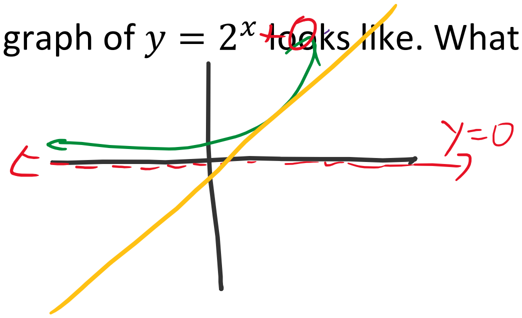
B) Simplify $\log_4(4^5) = 5$

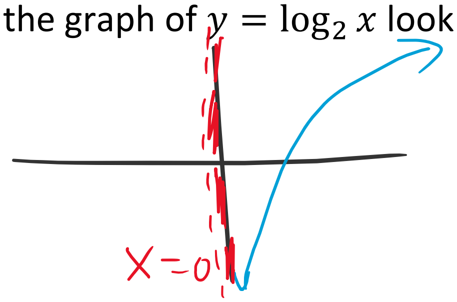
$4^5 = 4^5$

C) Simplify $3^{\log_3 6} = 6$

D) Simplify $e^{2\ln 8} = e^{2\log_e 8} = e^{\log_e(8^2)} = 8^2 = 64$

E) We know what the graph of $y = 2^x + 0$ looks like. What does the graph of $y = \log_2 x$ look like?





More consequences

F) If the points $(2,6)$ and $(4,36)$ are on the graph of $y = a^x$, find the average rate of change of $y = \log_a x$ over the interval $[6,36]$.

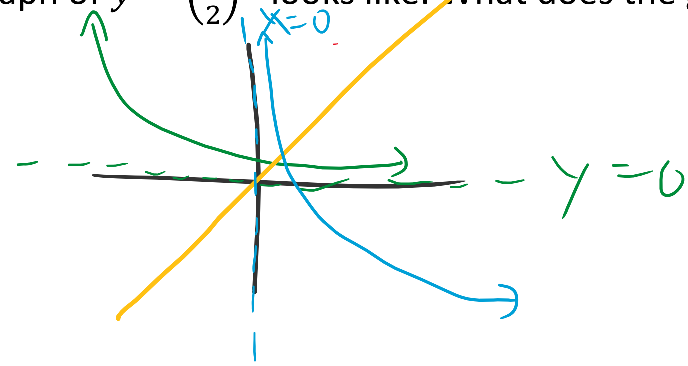
$(6,2)$ $(36,4)$

$$\frac{4-2}{36-6} = \frac{2}{30} = \frac{1}{15}$$

G) Simplify $\ln(e^3)$ $= 3$

H) Simplify $2^{\log_2 5 + \log_2 3}$ $= 2^{\log_2(5\cdot 3)} = 2^{\log_2 15} = 15$

I) We know what the graph of $y = \left(\frac{1}{2}\right)^x$ looks like. What does the graph of $y = \log_{\frac{1}{2}}(x)$ look like?



Two More

J) Simplify $8^{\log_2 x} = 8^{\frac{\log_8 x}{\log_8 2}} = 8^{\frac{\log_8 x}{1/3}} = 8^{3\log_8 x}$

$(2^3)^{\log_2 x} = 2^{3\log_2 x} = 2^{\log_2(x^3)} = x^3$ $= 8^{\log_8(x^3)}$

$= x^3$

K) $3^{\log_9 x} = (9^{\frac{1}{2}})^{\log_9 x} = 9^{\frac{1}{2}\log_9 x} = 9^{\log_9(x^{\frac{1}{2}})} = \sqrt{x}$

$3^{\frac{\log_3 x}{\log_3 9}} = 3^{\frac{\log_3 x}{2}} = 3^{\frac{1}{2}\log_3 x} = 3^{\log_3(x^{1/2})}$

$= \sqrt{x}$

Log Properties: Horizontal Dilations = Vertical Translations

Replace x with $\frac{x}{5}$

$$f(x) = \log_5 x$$

$$g(x) = \log_5 \left(\frac{x}{5}\right)$$

$g(x) = \log_5 x - \log_5 5$

Describe the transformation that maps the graph of f to the graph of g.

$g(x) = \log_5 x - 1$

horizontal dilation by a factor of 5

Vertical translation -1 units

Log Properties: Raising the Argument to a Power = Vertical Dilation

$$f(x) = \log_5 x$$

$$g(x) = \log_5(x^8)$$

$8\log_5(x)$

Describe the transformation that maps the graph of f to the graph of g.

Vertical dilation by factor of 8

Log Properties: All Log Functions are Vertical Dilations of Each Other

Rewrite the following with a base of 4

$f(x) = \log_2 x$ $\quad g(x) = \log_4 x$ $\quad h(x) = \log_8 x$ $\quad k(x) = \log_{16} x$

$f(x) = \dfrac{\log_4 x}{\log_4 2}$

$f(x) = \dfrac{\log_4 x}{1/2}$

$f(x) = 2\log_4 x$

$h(x) = \dfrac{\log_4 x}{\log_4 8}$

$h(x) = \dfrac{\log_4 x}{3/2}$

$h(x) = \frac{2}{3}\log_4 x$

$k(x) = \dfrac{\log_4 x}{\log_4 16}$

$k(x) = \dfrac{\log_4 x}{2}$

$k(x) = \frac{1}{2}\log_4 x$

Log Properties and Transformations Practice

Let $f(x) = \log x$

Match each of the numbered statements with the letter of the corresponding function.

#1) The graph of this function is the image of the graph of $f(x)$ after a vertical dilation by a factor of 10.

$10 \log x = \log(x^{10})$

#2) The graph of this function is the image of the graph of $f(x)$ after a vertical dilation by a factor of $\frac{1}{2}$.

$\frac{1}{2} \log x$

#3) The graph of this function is the image of the graph of $f(x)$ after a vertical translation by 1 unit.

#4) This function is the identity function.

A) $g(x) = \log(10x)$

$\log(10) + \log(x)$

$1 + \log x$

B) $g(x) = \log(10^x)$

$g(x) = x$

$g(5) = 5$

$g(4) = 4$

C) $g(x) = \log(x^{10})$

D) $g(x) = \log_{100} x$

$= \frac{\log x}{\log 100} = \frac{\log x}{2} = \frac{1}{2} \log x$

Recognizing Log Functions From Tables

The following is a table of values of
$f(x) = a(b)^x$

x	$f(x)$
-3	3
0	6
3	12
6	24
9	48

Let's make a table of values for $f^{-1}(x)$

x	$f^{-1}(x)$
3	-3
6	0
12	3
24	6
48	9

Exponential functions: input has constant difference ⇔ output has constant ratio/proportion

Log functions: input has constant ratio/proportion ⇔ output has constant difference

One of these tables is from an exponential function, one of these tables is from a logarithmic function, and one of these tables is a distractor. Which is which?

exponential

x	$f(x)$
2	48
4	72
6	108
8	162
10	243

+2 +2 +2 +2

$\cdot\frac{3}{2}$ $\cdot\frac{3}{2}$ $\cdot\frac{3}{2}$ $\cdot\frac{3}{2}$

x	$g(x)$
1	120
4	60
16	30
64	15
256	7.5

logarithmic

x	$h(x)$
2	25
10	20
50	15
250	10
1250	5

$\cdot 5$ $\cdot 5$ $\cdot 5$ $\cdot 5$

-5 -5 -5 -5

$$\frac{72}{48} = \frac{36}{24} = \frac{9}{6} = \frac{3}{2}$$

$$\frac{108}{72} = \frac{54}{36} = \frac{6}{4} = \frac{3}{2}$$

$$\frac{162}{108} = \frac{81}{54} = \frac{27}{18} = \frac{3}{2}$$

Let $f(x) = 2^x - 5$

Find an expression for $f^{-1}(x)$

Let $g(x) = \log_3(x-8)$

$y = \log_3(x-8)$

Find an expression for $g^{-1}(x)$

$3^{(x)} = 3^{(\log_3(y-8))}$

$3^x = y-8$

$3^x + 8 = y$

$g^{-1}(x) = 3^x + 8$

Find the inverse of each of the following functions

$f(x) = 2(3)^{x-4}$

$g(x) = \ln(x) + 8$